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# TAILORED NONLINEAR NEGATIVE STIFFNESS MECHANISMS FOR LINEAR MOTION

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**Key words:** Negative stiffness mechanism; Linear motion; Kinematics tailoring; Energy balancing; Actuator efficiency.

**Abstract.** Traditional ways to achieve the desired motion of mechanisms or deformation of morphing structures require external energy for actuation. Frequently the use of these actuators to drive the system can cost noteworthy energy for each cycle of operation and the spent energy cannot be recovered. This work investigates a passive energy balancing concept for linear motion systems by strategically using a negative stiffness mechanism. The energy balance concept is achieved by employing a negative stiffness system to couple with the positive stiffness mechanical system to create zero stiffness which can be driven with smaller actuators. The negative stiffness mechanism proposed here uses a pre-tensioned spring to produce a passive torque and therefore to transfer the passive torque through a crankshaft for linear motion. The kinematics of the negative stiffness mechanism is first developed to satisfy the required linear motion and its geometry is then optimised to achieve minimal energy requirements. The performance of the optimised negative stiffness mechanism is evaluated through the net force and the total required energy. Exploiting the negative stiffness mechanism has a significant benefit in the field of energy sensitive applications.

## 1 INTRODUCTION

Traditional mechanisms and smart structures are designed by using actuators or smart materials to overcome the internal (e.g. structure deformation) resistance and external (e.g. aerodynamic) loads to generate motion of the system. These integrated systems allow shape changes of the structure, and thus control its motion with acceptable precision. In particular, morphing requires deformation against the internal loads, and the frequent use the actuators to drive the system can cost noteworthy energy for each cycle of operation and the spent energy cannot be recovered. In other words, the work done to reach the target state requires the input of energy, while the subsequent dissipation of that energy to recover the initial state generates waste heat. Traditional actuators, such as electromechanical actuation, can be used as linear and rotary actuators, combined with mechanisms to provide a powerful tool for morphing [1]. Moreover, with the development of smart materials, many traditional actuation systems based on electric motors, hydraulics, or pneumatics may be replaced by smart material systems. For example, piezoelectric materials have been used as actuators to control wing panels [2,3], spanwise deflection [4] and trailing-edge flaps [5,6]. Concepts have been proposed to reduce the required force or torque and therefore to allow smaller and lighter actuators, or even no actuator, to be employed to drive the system. For some typical cases, such as gravity equilibrators, a weight can be carried throughout its range of motion without any external energy by using spring mechanisms or counterweights [7–9]. Such static balancing is described as "zero stiffness" or "neutral stability", as the system can be moved without operating energy. In general, many systems have force that is purely a function of displacement, as shown in Fig. 1(a), known as an elastic force response. In order to produce a static balancing for such systems with a zero stiffness property, negative stiffness is required to assist the imposed deformation. Figure 1(b) shows a representative nonlinear stiffness property, which has a completely nonlinear behaviour compared to the positive stiffness system and helps to construct an energy balancing system in a certain working range shown in Fig. 1(c). As the actuation is returned to zero, the elastic energy stored in the driven structure will force the negative stiffness system back to its original state, again at zero net actuation force. Therefore, Figure 1 shows that for a system assumed to be perfectly elastic, the negative stiffness can drive required actuation force to zero. Of course, many real systems include significant inelastic forces, which can be rate or history dependent, or due to external disturbances, and these cases represent a more complex problem that will be the subject of future work. In what follows, it is assumed that the driven structure has a purely elastic force displacement response.

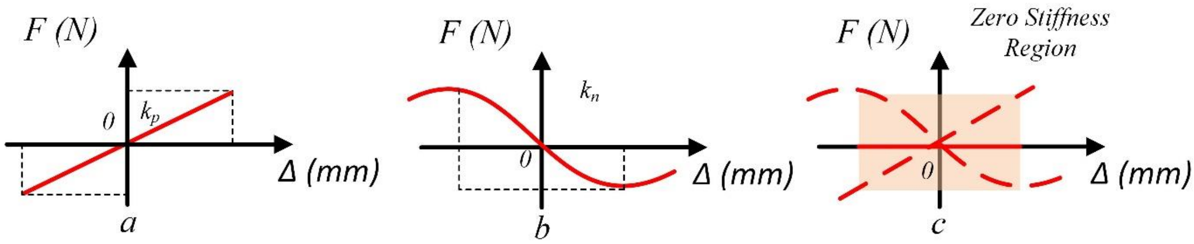


**Figure 1.** Schematic of the energy balancing system force curve. $k_p$ is the stiffness of the load system and $k_n$ is the negative stiffness system. (a) Increasing force system. (b) Nonlinear stiffness system. (c) Energy balancing system in a certain working range.

The negative stiffness mechanism can be coupled to the target positive stiffness system to produce an energy balancing system and the energy required to actuate the existing system can be balanced by the stored energy in the negative stiffness system. Figure 2 shows that a net zero stiffness device may be achieved by adding the negative stiffness system to a positive stiffness system, and the coupled system can therefore be considered as an energy balancing system. Figure 2(a) shows that a traditional way to actuate a system with internal stiffness $k_p$ and the external load $L$, which requires a high power actuator. However, by coupling the negative stiffness mechanism, the total stiffness of the whole system becomes $k_p + k_n$ and a key benefit is that a small actuator can be used to determine the same output with regards to the same external load $L$, as shown in Fig. 2(b).

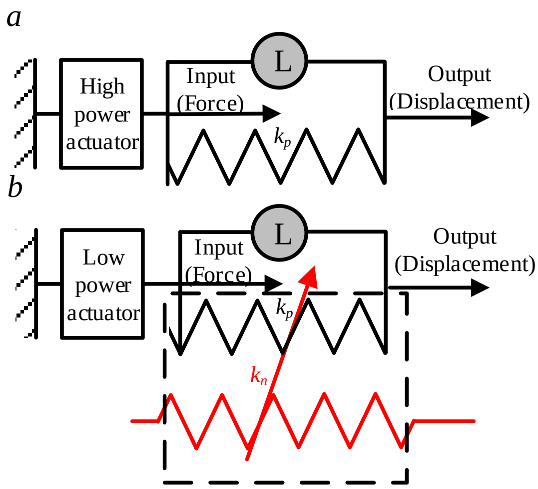


**Figure 2.** Minimalistic model of a negative stiffness mechanism for energy balancing. $L$ denotes the external load, $k_p$ is the internal stiffness of the load system and $k_n$ is the negative stiffness system. (a) Traditional actuated system. (b) Energy balancing system by adding the negative stiffness mechanism.

It can be assumed that if the required input, such as force or torque, can be completely eliminated by the negative stiffness system, then in principle no energy is required to move the system, other than to overcome dissipation. Therefore, the use of negative stiffness systems is likely to benefit energy constrained systems, such as those present in the aerospace and automotive industries.

Many applications have been proposed that use a negative stiffness for passive energy balancing. Clingman and Ruggeri [10] described a negative stiffness nonlinear over-centre linkage used on a tilt-rotor blade for active twist. This negative stiffness linkage mechanism uses the stored energy of a compressed spring to rotate the output shaft, resulting in an effectively softened blade that requires 70% reduction in torque for morphing. In addition, a pulley based balancer has been proposed with a varying radius pulley to preserve moment equilibrium between a constant load and a varying spring length [11,12]. By using such a spiral pulley negative stiffness mechanism, the required torque for the morphing actuation can be satisfactorily matched to create a nearly zero stiffness which requires minimal energy. A bidirectional torsional negative stiffness mechanism by using a series of pre-compressed springs has investigated for energy balancing systems [13]. This integrated BTNS mechanism was then verified to tailor the kinematics of the required torque driving the active camber and the results showed a similar torque-rotation profile can be generated. The proposed device provides actuation to change the state of the system, such as deforming a structure or lifting a mass; hence the energy provided by the actuator transforms into an increased potential energy in the system. Once the system returns to its original state, all of the energy provided by the actuator will be recovered, if the system is conservative. In other words, the work done to reach the target state require the input of energy, while the subsequent dissipation of that energy to recover the initial state generates waste heat.

In a previous work [14], an energy efficient concept for bidirectional morphing aircraft actuation was investigated by using a negative stiffness mechanism. The torsional negative stiffness with an off centre spring (TNSOCS) mechanism was proposed that uses a pre-tensioned spring to convert the decreasing spring force available in the spring into an increasing output balanced torque. A significant contribution can be provided by the negative stiffness mechanism to balance the positive stiffness rotation motion system. This work extends [14] by applying the torsional negative stiffness mechanism to a linear motion actuation. This paper proceeds as follows. Firstly, a prototype of this energy balancing device is proposed and the kinematics of the bidirectional torque shaft, the linear motion mechanism and rotation angle are investigated. Then, the characteristics of the negative stiffness mechanism is proposed and a linear spring is chosen as the positive stiffness load system for investigation. Finally, the design parameters are optimised to exactly match the requirements of the linear motion requirement.

## 2 KINEMATIC ANALYSIS

The prototype of this energy balancing device is shown in Fig. 3 with the detailed geometry definition of two mechanisms. The torsional negative stiffness with an off centre spring (TNSOCS) mechanism [14] is developed to produce the actuating force and the slider-crank mechanism is designed to transfer the torque to linear motion actuation. The shaft is centred at $O$ and $\theta$ is the shaft rotation angle. A pre-stretched spring is fixed on the shaft and point $B$ is the off centre point for the spring. Then, the slider-crank mechanism is connected to the torsional negative stiffness mechanism and point $E$ is the joint. Therefore, when the pulley rotates, the slider can move in the horizontal direction as denoted by $q_m$.

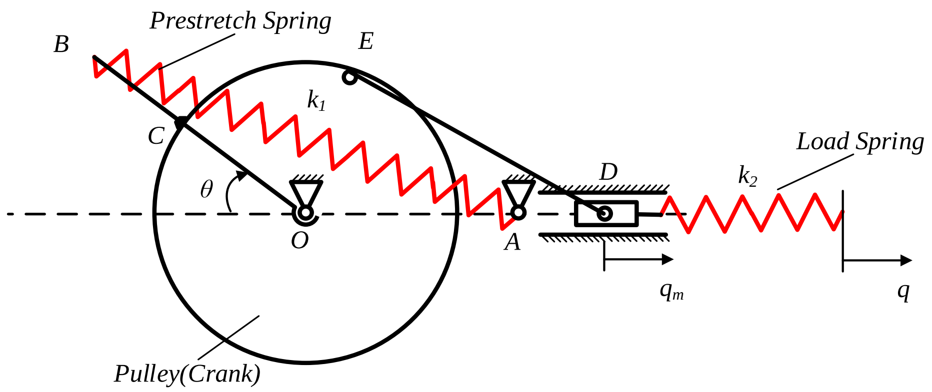


**Figure 3.** Schematic representation of a variable stiffness cam mechanism. In this mechanism $k$ denotes the stiffness of the linear spring, $q$ is the extra output position, $\theta$ is the input angle, while $q_m$ denotes the motion of the slider.

In order to investigate the tailored nonlinear negative stiffness mechanism for linear motion, the kinematics of the system should be derived. Two springs are shown here, one is designed as an energy storage device for the negative stiffness system and the other represents the load system. Therefore, the potential energy function of this mechanism is given by:

$$V(\theta, q) = \frac{1}{2}k_1(l - l_0)^2 + \frac{1}{2}k_2(q_m - q)^2 \tag{1}$$

where $l_0$ is the initial length of the spring and $l$ is the length of the spring at its current position. $k_2$ is the stiffness of the output system and can be considered as a linear system when $k_2$ is constant. Then, the additional required moment imposed by the negative stiffness modulating mechanism is

$$M_{in}(\theta, q) = \frac{\partial V}{\partial \theta} = k_1(l - l_0)\frac{\partial l}{\partial \theta} + k_2(q_m - q)\frac{\partial q_m}{\partial \theta} \tag{2}$$

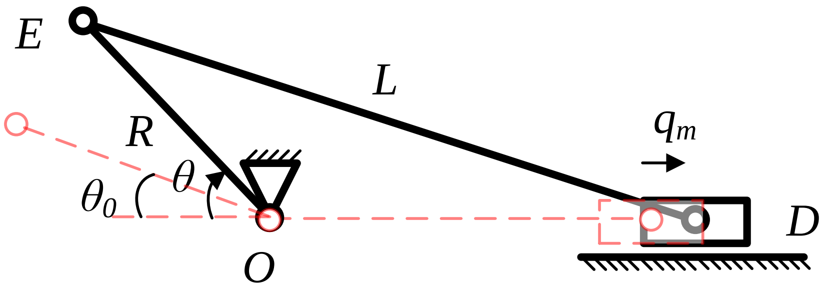


**Figure 4.** The slider-crank mechanism: displacement $q_m$ of the slider for different $\lambda$ and $\theta$.

Moreover, the displacement $q_m$ of the slider can be determined as

$$q_m = L\left(\left(\sqrt{1 - \lambda^2 \sin^2\theta} - 1\right) + \lambda(1 - \cos\theta)\right) \tag{3}$$

where $\lambda = R/L$ denotes the ratio of the radius of the crank shaft and the length of the connecting rod. Figure 5 shows the displacement of the slider for different $\lambda$ when the crank shaft rotates. It can be seen that the larger the value of $\lambda$, the higher the displacement that can be achieved.

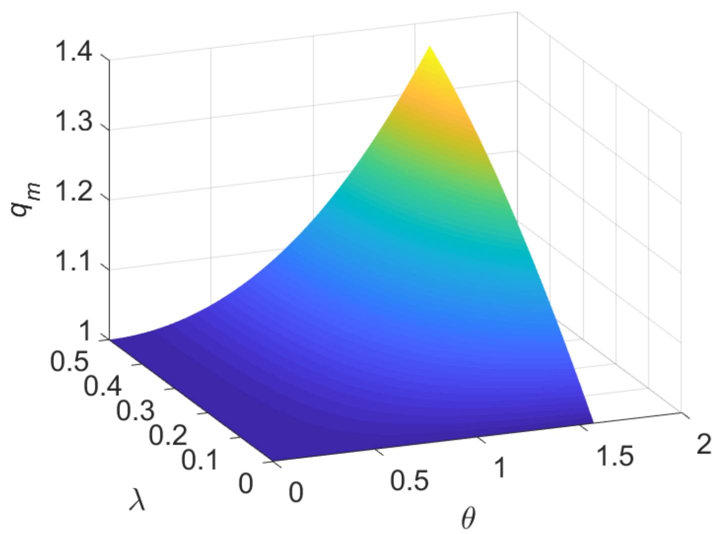


**Figure 5.** Displacement $q_m$ for different $\lambda$ and θ .

The kinematics of the torsional negative stiffness shaft is now investigated. Figure 6 shows that the cable is fixed on the shaft and point $B$ is the off centre point for the spring. The Cartesian coordinates can then be defined with the origin at $A$ and the coordinates of any point $B'$ are then defined as

$$x_B = -l_{off} - b\cos\theta \tag{4}$$

$$y_B = b\sin\theta \tag{5}$$

where $r$ is the length of the vector $\overline{OB}$ and $l_{off}$ is the length of the vector $\overline{OA}$.

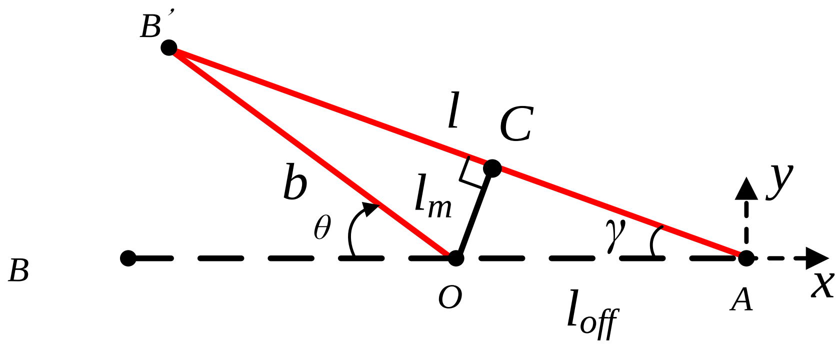


**Figure 6.** Torsional negative stiffness shaft geometry analysis with moment arm details.

Therefore, the length of the vector $\overline{AB'}$ is equal to

$$l = \sqrt{x_B^2 + y_B^2} = \sqrt{(b\sin\theta)^2 + \left(l_{off} + b\cos\theta\right)^2} \tag{6}$$

The change of the length $l$ with the shaft rotation angle $\theta$ can be calculated as

$$\frac{\partial l}{\partial \theta} = -l_{off}\, b\sin\theta \Big/ \sqrt{(b\sin\theta)^2 + \left(l_{off} + b\cos\theta\right)^2} \tag{7}$$

The motion of the slider can also be calculated as

$$\frac{\partial q_m}{\partial \theta} = L\left(\lambda\sin\theta - \lambda^2\sin\theta\cos\theta \Big/ \left(\sqrt{1-\lambda^2\sin^2\theta}\right)\right) \tag{8}$$

Therefore, the additional required moment can be obtained by substituting Eqs. (7) and (8) into Eq. (2), to give

$$\begin{aligned} M_{in}(\theta,q) = \frac{\partial V}{\partial \theta} = k_1 &\left(\sqrt{(b\sin\theta)^2 + \left(l_{off} + b\cos\theta\right)^2} - l_0\right) \\ &\times \left(-l_{off} b\sin\theta \Big/ \sqrt{(b\sin\theta)^2 + \left(l_{off} + b\cos\theta\right)^2}\right) \\ &+ k_2 L^2 \left(\left(\sqrt{1-\lambda^2\sin^2\theta} - 1\right) + \lambda(1-\cos\theta) - q\right) \\ &\times \left(\lambda\sin\theta - \lambda^2\sin\theta\cos\theta \Big/ \sqrt{1-\lambda^2\sin^2\theta}\right) \end{aligned} \tag{9}$$

In Eq.(9), for an ideal energy balancing system, we require $M_{in}(\theta,q) = 0$, where $k_1$, $b$, $l_{off}$, $L$, $l_0$, $q$, $\lambda$, $k_2$ are system constants. For a potential actuation system with known $k_2$ and $q$, the parameters can be optimised for passive energy balancing. Moreover, Eq. (9) can be modified by employing $k_1 = 0$ and $q = 0$ to obtain the ordinary required moment to actuate the system without the proposed negative stiffness mechanism as

$$M_r = k_2 L^2 \left(\left(\sqrt{1-\lambda^2 \sin^2\theta} - 1\right) + \lambda(1-\cos\theta)\right) \times \left(\lambda \sin\theta - \lambda^2 \sin\theta \cos\theta / \left(\sqrt{1-\lambda^2 \sin^2\theta}\right)\right) \tag{11}$$

Figure 7 shows the required moment and actuation force for the different input angle, and demonstrates that the required moment has a strong nonlinearity. Moreover, the system will be turned from positive stiffness to negative stiffness at a specific angle.

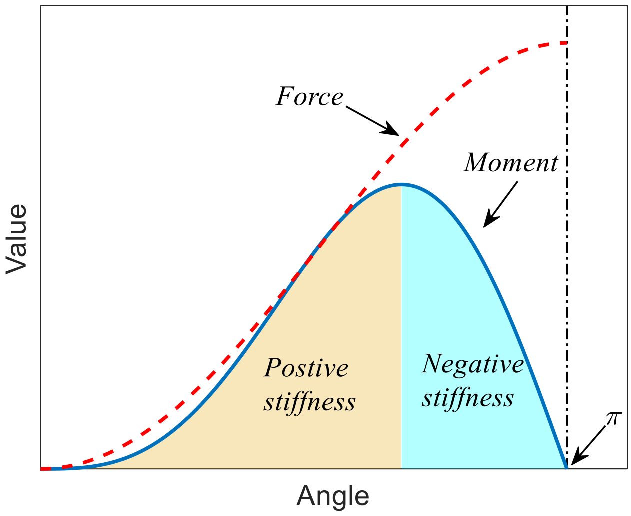


**Figure 7.** Required moment and actuation force as a function of input angle.

Therefore, in order to improve the performance of the total system, an initial established angle $\theta_0$ of the bar mechanism is proposed as an additional parameter, as shown in Fig. 4. Then, the displacement $q_m$ of the slider becomes

$$q_m = L\left(\left(\sqrt{1-\lambda^2 \sin^2(\theta+\theta_0)} - 1\right) + \lambda(1-\cos(\theta+\theta_0))\right) \tag{12}$$

Moreover, the turning point $\theta_t$ of the system can be obtained by solving

$$\frac{\partial M_r}{\partial \theta} = 0 \tag{13}$$

In order to evaluate the performance of the energy balancing system, it is useful to establish a force metric. As the system is continuous with limited constants, it is hard to construct a perfect energy balancing system using finite unknown constants. Therefore, an error metric is established to evaluate the performance of the energy balancing system. Figure 8 shows the schematic diagram of the actuation force error for different input angles. The error metric could theoretically integrate this actuation error from a zero angle to the current position and is written as

$$\eta_{err} = \int_0^\theta |M_{in}|\, d\theta \tag{14}$$

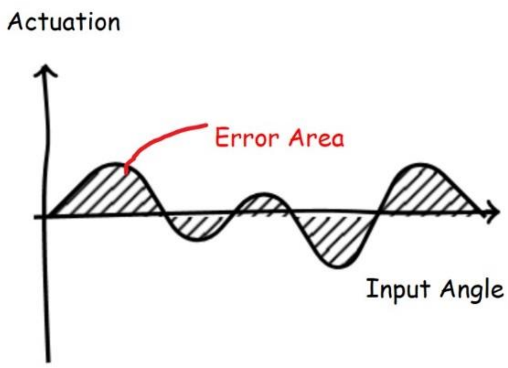


**Figure 8.** Schematic diagram of the actuation force error for the input angle.

This is a meaningful metric to evaluate the error for the passive energy balancing (PEB) system, as this implies the additional energy required during actuation. With this actuation force error metric, the performance of the energy balancing system can be obtained, and the geometric parameters of the system can be optimized using the *fmincon* function in Matlab. The energy conversion efficiency metric $\eta_{err}$ is used as the objective function to make the torque available from the negative stiffness mechanism match as closely as possible the requirement to actuate the linear motion system over the entire prescribed rotation range.

## 3 DESIGN CASE

In order to investigate how to use this negative stiffness mechanism for linear motion actuation, a linear spring stiffness $k_2 = 69$ N/m is chosen as the drive load for study and the rotation input angle is between 0 and $\theta_t - \theta_0$. The resulting optimised parameters are shown in Table 1. All the components used here are ubiquitous and already manufactured over a very broad range of scales. The intention of this initial study is therefore to show the considerable energy storage achievable in a moderate size demonstrator.

**Table 1.** Design parameters for the NS mechanism for linear motion optimisation.

| Parameter | Lower bound | Upper bound | Optimised value | Units |
|---|---|---|---|---|
| Drive spring rate, $k_1$ | 0.01 | 1 | 0.0238 | N/mm |
| Drive spring length, $l_0$ | 10 | 500 | 146.186 | mm |
| Off centre length, $l_{off}$ | 10 | 500 | 124.270 | mm |
| Link bar length, $L$ | 10 | 500 | 100.000 | mm |
| Shaft length, $b$ | 10 | 500 | 125.146 | mm |
| Ratio $R/L$, $\lambda$ | 0.1 | 1 | 0.2709 | - |
| Initial established angle, $\theta_0$ | 0 | 90 | 78.9890 | degree |

With the optimised parameters shown in Table 1, the effectiveness of the optimal negative stiffness mechanism to construct an energy balancing system for the proposed target positive stiffness linear motion spring system can be investigated. Figure 9 shows the linear motion displacement along with the rotation angle provided by the negative stiffness mechanism. While there is a strong nonlinearity in the moment function, the output displacement presents a linear relationship with respect to the rotation during the optimised operation interval, which can also be seen in Fig. 7. Hence this characteristic implies that the negative stiffness mechanism can provide a suitable linear motion output by careful design and optimisation.

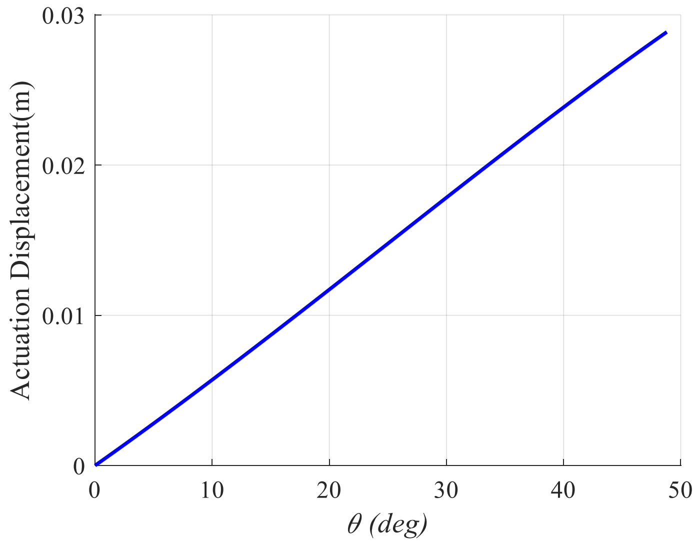


Figure 9. Predicted actuation displacement with rotation.

The performance predicted for the optimised torque shaft profile shows satisfactory matching of the linearised torque requirements. Figure 10 shows that the evolution of torque with rotation for the spring and the linear motion system and the net torque of the whole system. The torque provided by the negative mechanism matches the torque required closely to provide a linear motion, and the maximum torque required by the extra actuator is approximately zero.

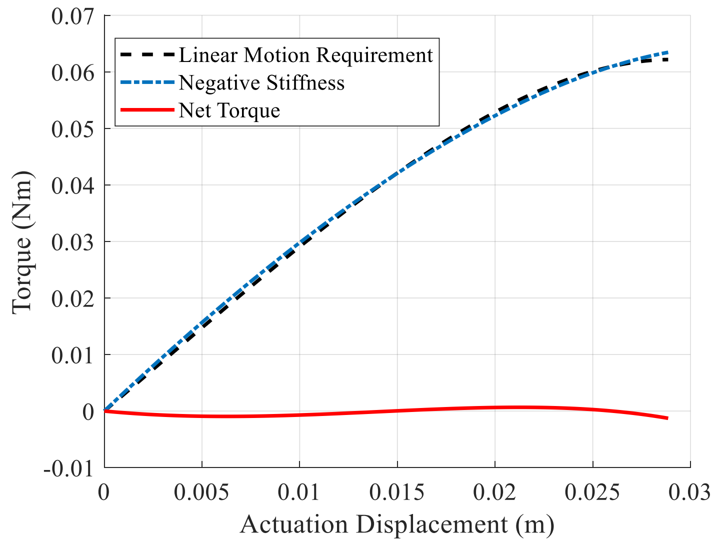


Figure 10. Predicted torque with optimised negative stiffness mechanism.

Integrating both the torque versus the rotation angle provides the mechanical energy required to actuate the target spring, and is shown in Fig. 11. By comparing the energy required with and without the negative stiffness mechanism, the negative stiffness mechanism shows a strong ability to passively balance the required torque. Figure 11 shows that the predicted energy reduction is almost 98%, with the energy required reduced from 0.032 J to 0.0005 J. This is the contribution of the energy stored in the extended spring in the initial position.

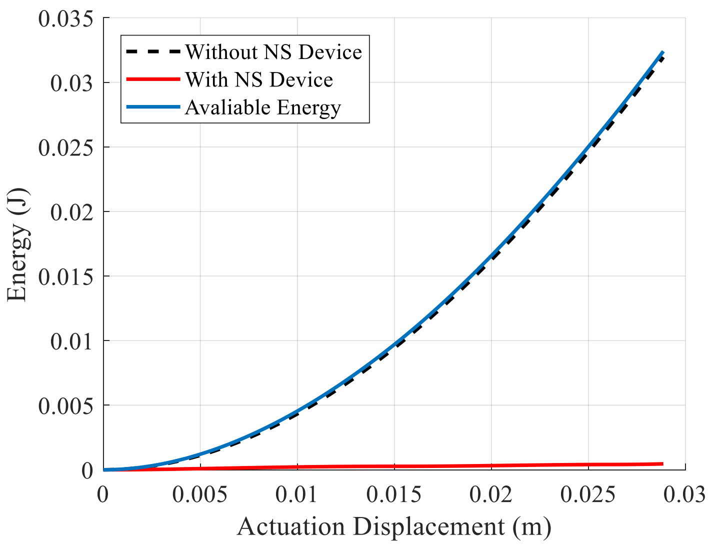


Figure 11. Comparison of the predicted energy required with and without the negative stiffness mechanism.

The objective error function plotted in Fig. 12 shows that the optimised configuration of the negative stiffness mechanism provides significant benefits in terms of energy efficiency.

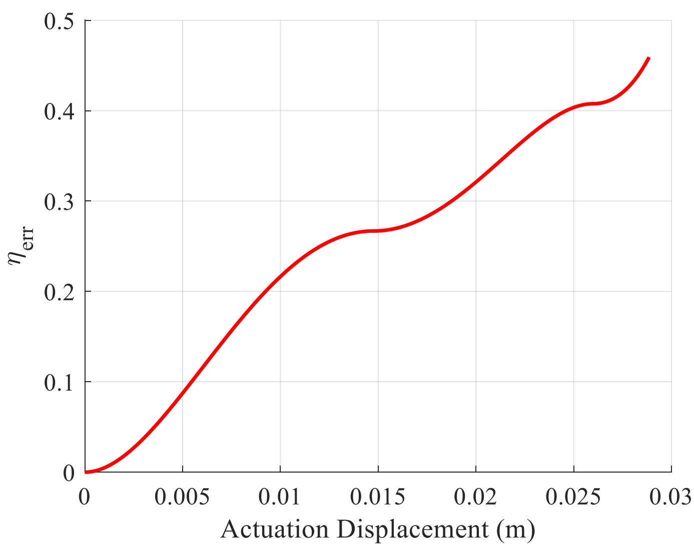


Figure 12. Evolution of efficiency with rotation.

Based on this study, it can be seen that the negative stiffness mechanism provides a significant contribution to balancing the positive stiffness system, and can provide a suitable linear motion output by careful design and optimisation. High energy conversion efficiency can be provided by the extension spring and a smaller actuator can be used in the system.

## 4 CONCLUSION

A new concept using a torsional negative stiffness with an off centre spring (TNSOCS) mechanism for linear motion actuation has been presented. The proposed negative stiffness device is designed to have the capability to passively balance the actuation requirement of systems displaying positive stiffness. The motivation for using such a mechanism was presented and the kinematics of the tailored nonlinear negative stiffness mechanism was introduced. An error metric function was introduced to provide a basis for evaluation and also act as the objective function to optimise the geometry of the tailored nonlinear negative stiffness mechanism. The optimised tailored nonlinear negative stiffness mechanism was shown to be able to generate a torque that matches the required force of a linear spring closely, which means a significant contribution can be provided by the tailored nonlinear negative stiffness mechanism to convert the torque to linear actuation and balance the positive stiffness system. The concept shown here can be extended to many potential actuation design applications which need frequent state switching to reduce the mean power consumption and waste heat dissipation. While the example used in this paper is relatively simple, it provides an insight into the low energy actuation design problem, both in the field of bidirectional morphing aircraft and other fields.

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